October 16, 2024

Notes for Meeting 6

Outline:

Recap from Meeting 5

I. Reasons and Logic: Semantic Logicism and Logical Expressivism
II. Sequent Calculus Metavocabularies for Expressivist Logic: Gentzen and NMMS
III. Trilogics K3 and LP from an Expressivist Perspective
IV. Four examples of distinctions-with-relations emerging from relaxation of structure.

Recap:

The structure of material reason relations.
Reason Relations, Two Kinds of Closure, Two Kinds of Openness.
A few points: rejecting MO and transitivity are closely related. (Simonelli shows how to turn nonmonotonicity examples into nontransitivity examples, and Ulf shows that given our minimal expressivist requirement on conditionals, CO and CT entail MO.)
Explicitation closure and its denial are new topics.
Embracing *Rational Hysteresis* as opposed to "rational closure" under minimal CM and CT.

Argued that there are many potentially interesting *formal* issues that arise for reason relations, issues of structure that can at least potentially be treated in a mathematical vocabulary, that come up before, and independently (as far as anything I said last time is concerned) of any specifically *logical* vocabulary, concepts, principles, or systems.

This raises the question: so where *does* logic fit in, in thinking about reasons, in the sense of reason relations?

I. Reasons and Logic: Semantic Logicism and Logical Expressivism

a) Logicism about the relations between *reasons* and *logic* is the view that good reasons are, in the end, always *logically* good reasons, articulated by deductive logical relations of implication and inconsistency.

b) Expressivism about the relations between *reasons* and *logic* is the view that the distinctive job of logical vocabulary is an expressive one, to make explicit in claimable form antecedent material reason relations of implication and incompatibility.

c) Point is Socratic and critical: to bring reason relations into the game they govern, as claimables that can be rationally challenged and require rational defense. Before logical vocabulary is introduced, practitioners can be critical about *doxastic* commitments, assessing their entitlements by the pragmatic structure of default-challenge-defense. Introducing logical vocabulary brings into that practice ("game") the reason relations that normatively govern that minimal discursive practice. It brings reason relations into discrsive practices in the form of claimables, which can be rationally challenged, by reasons against them, and rationally defended, by reasons for them. So reason relations become subject to crtical rational scrutiny and development.

d) Logic as the organ of *rational self-consciousness*. For semantic inferentialists (or as far as semantic content is specifiable in an inferentialist MV), logic as the organ of *semantic* self-consciousness.

e) But McD objects to layercake picture.
"The light dawns slowly over the whole."
He denies that it is intelligible that some discursive practices— we can agree on these being ones in which it is possible to claim that p, that things are thus-and-so—do not support critical assassment of the goodness of candidate implications and incompatibilities.
I think it is intelligible that practitioners can be critical (we agree that the must be critical to be rational—and critical assessment is governed by norms in the form of both kinds of reason relations, since one must assess whether any of the consequences of some claimables are incompatible with some other claimables) Just about commitments to accept or reject claimables, without yet being able to extend that critical gaze to the rules of claiming, in the form of reason relations.

f) In philosophy of logic, this is an answer to the ***demarcation* question** (what makes something a logical concept, or a bit of logical vocabulary). It makes the *correctness* question inappropriate. (Distinguish *conditionals:* two-valued, intuitionistic, strict implication.)
Notice that then the traditional paired ***correctness* question** lapses.

According to expressivists, conditionals must *codify* (in the sense of DD) some notion of <u>good implication</u>, appealed to in he turnstile of DD: $\Gamma|\!\sim A\rightarrow B$ iff $\Gamma, A|\!\sim B$. But there are *many* possible senses of "good implication", some of which are potentially interesting. So:

i. *Two-valued horseshoe* codifies the (minimal)sense of "good implication" according to which an implication is good if it does *not* have true premises and a false conclusion. Notice that the Restall bilateral reading (and its truthmaker analogue) is an intensional version of this classical minimal extensional idea.

ii. *Intuitionistic conditionals* codify the sense of "good implication" according to which an implication is good just in case there is a procedure (of a certain sort, where specifying and justifying the choice of this sort is a crucial philosophical task) for turning a defense (they say: "proof", but the idea can be applied more broadly, so that this insiistence on monotonic dispositiveness is seen as just one specific form requirements like this could take) of the premises (a reason implying them) into a defense of the conclusion.

iii. C. I. Lewis's *hook of strict implication* codifies the sense of "good implication" according to which an implication is good if it is impossible for its premises to be true and its conclusion false. Note that the Hlobil definition of consequence in the TM framework is a kind of application of this idea in a more intensional semantic MV. These alternatives illustrate that many conceptions of reason relations are possible.

g) Logical vocabulary as extending a base vocabulary, rather than as a free-standing vocabulary.

h) The expressive power of logical vocabulary, the coditional, construed in any of these ways is, from our point of view, that the build in structural restrictions that preclude their applicability to many potentially interesting base Vs.

i) Expressivist criteria of demarcation/adequacy:

The reason relations governing sentences containing logical vocabulary must be

i) *El*aborated from and

ii) *Ex*plicative of

(Shorthand: "LX for") the *reason relations* of a *non*logical (material) *base* vocabulary.
For L: Must be able to *conservatively* extend the base vocabulary to *compute* the reason relations of the logically extended vocabulary from the reason relations of the underlying material base vocabulary.

j) Paradigm of what counts as making reason relations explicit as claimables:
DD and II (for genuinely expressive, as opposed to merely aggregative connectives):

• Deduction-Detachment (DD) Condition on Conditionals:

$\Gamma |\!\sim A \rightarrow B$ if and only if $\Gamma, A |\!\sim B$.

• Incoherence-Incompatibility (II) Condition on Negation:

$\Gamma |\!\sim \neg A$ if and only if $\Gamma \# A$, i.e. $\Gamma, A |\!\sim$.

These conditions cause two problems in the context of sequent calculi:

1. An obvious structural mismatch: DD and II are *bi*conditionals. But sequent rules must always *complicate*, adding structure. So they are essentially *a*symmetric.
2. Ulf's argument:

A basic constraint on such a construction is set out by a simple argument due to Ulf Hlobil. He realized that in the context of Contexted Reflexivity and a Ramsey conditional, Cut entails Monotonicity. For if we start with some arbitrary implication Γ|~ A, we can derive Γ,B|~ A for arbitrary B— that is, we can show that arbitrary additions to the premise-set, arbitrary weakenings of the implication, preserves those implications. And that is just monotonicity. For we can argue:

a)	Γ|~ A	Assumption
b)	Γ, A, B|~ A	Contexted Reflexivity (CO)
c)	Γ,A|~ B → A	Ramsey Condition Right-to-Left
d)	Γ|~ B → A	Cut, Cutting A using Assumption
e)	Γ, B|~ A	Ramsey Condition Left-to-Right.

Since we want to explore adding Ramsey conditionals to codify material implication relations that are reflexive but do not satisfy Cut—so that prelogical explicitation is not treated as always inconsequential—we will sacrifice Cut in the logical extension.

II. Sequent Calculus Metavocabularies for Expressivist Logic: Gentzen and NMMS

Explain sequent calculi:
Take **sequents as objects** (represented by pairs of sets of sentences).
Sequents are thought of as **expressing/representing reason relations** (of both kinds, by a philosophically misleading notational-coding convenience-trick).
A sequent calculus is a set of meta-inferential principles, that say (of the form):
If these sequents hold (are distinguished, get a ribbon), then so are these others.
Those principles are formulated in a distinctive vocabulary. It is a vocabulary for specifying not only reason relations, but consequential (reason) relations among reason relations (represented as sequents).
The metainferential principles formulated in the SC metavocabulary are of two kinds: structural rules and connective definitions.

A key point of connective definitions is that they let one compute the reason relations of sentences containing logical connectives from the reason relations of sentences that do not.

SC codifies a function from reason relations of one vocabulary to reason relations of a supervocabulary of it, in the sense that one lexicon is a subset of the other, and the reason relations of one are a subset of those of the other.

'Sequent' is literally "what follows." Sequents specify what (sets of sentences) follow from (or, by the potentially misleading coding trick, are incompatible with) which others.
The metainferential rules specifiable in the SC metavocabulary are of the form:
Whenever set X of sequents is distinguished, so is set Y.
That is marked by a horizontal line.
That line, marking the consequence relation among (sets of) sequents, that is, among sets of reason relations, is transitive, and it is monotonic. (Throwing in some additional sequents above the line never infirms the derivability below the line of anything derivable from any subset of what is above the line.) So the *use* vocabulary of SC is governed by reason relations that satisfy traditional topological closure structural restrictions.

As to "distinguished": This apparatus can be used abstractly (put an asterisk on the sequent) to express the preservation of *any* sort of status that sequents can have (e.g. pleasingness to God).
So we can ask: **what statuses can sequents have that would be interesting to elaborate by SC principles?**
One prime candidate is sequents codifying reason relations that actually hold.
We have given two senses of that, two things the distinguishing asterisk marking what SC rules preserve can mean when $\Gamma|\sim\Delta$ gets distinguished:

i) That the position consisting of commitment to accept all of Γ and reject all of Δ is “out of bounds”: one to which one is precluded from being entitled.

ii) That every fusion of any exact truthmaker of all of Γ with any exact falsemaker of all of Δ is an impossible state.

And we have shown how these two statuses distinguishing sequents (reason relations) can be isomorphic, that is, distinguish-by-endorsement just the same reason relations.

SC specifies consequences, for reason relations, of other reason relations.
That capacity raises the question of what it would be illuminating to apply that apparatus to. That is, what set of reason relations endorsed or distinguished in whatever way we are interested in. (Could be the truth-preserving ones, or those pleasing to God, and all sorts of other possibilities of intermediate interest.)
One choice is structural “tautologies”. Gentzen uses instances of RE for logical atoms. This yields pure logic. Our NMMS yields classical logic when applied to such a flat base vocabulary.
Pure logic is just what you get from a flat prior, in a sense that belongs in an important genus with the bayesian sense of that term.
but we think it is much more interesting to look at what happens when an SC is applied to a substantive base vocabulary, i.e. reason relations, that is, when it is applied to substantive “priors”.
Since we want our SC to introduce logical vocabulary that will be as universally LX as possible, we care about applicability to base vocabularies that do not meet traditional structural constraints.

LK (Gentzen):

L→: $$\frac{\Gamma \mid\!\sim \Delta, A \quad \Gamma, B \mid\!\sim \Delta}{\Gamma, A \to B \mid\!\sim \Delta}$$

R→: $$\frac{\Gamma, A \mid\!\sim \Delta, B}{\Gamma \mid\!\sim \Delta, A \to B}$$

L¬: $$\frac{\Gamma \mid\!\sim \Delta, A}{\Gamma, \neg A \mid\!\sim \Delta}$$

R¬: $$\frac{\Gamma, A \mid\!\sim \Delta}{\Gamma \mid\!\sim \Delta, \neg A}$$

L∧: $$\frac{\Gamma, A, B \mid\!\sim \Delta}{\Gamma, A \wedge B \mid\!\sim \Delta}$$

R∧: $$\frac{\Gamma \mid\!\sim \Delta, A \quad \Gamma \mid\!\sim \Delta, B}{\Gamma \mid\!\sim \Delta, A \wedge B}$$

L∨: $$\frac{\Gamma, A \mid\!\sim \Delta \quad \Gamma, B \mid\!\sim \Delta}{\Gamma, A \vee B \mid\!\sim \Delta}$$

R∨: $$\frac{\Gamma, \mid\!\sim \Delta, A, B}{\Gamma \mid\!\sim \Delta, A \vee B}$$

NMMS:

L→: $$\frac{\Gamma \mid\sim \Delta, A \quad \Gamma, B \mid\sim \Delta \quad \Gamma, B \mid\sim \Delta, A}{\Gamma, A \rightarrow B \mid\sim \Delta}$$

R→: $$\frac{\Gamma, A \mid\sim \Delta, B}{\Gamma \mid\sim \Delta, A \rightarrow B}$$

L¬: $$\frac{\Gamma \mid\sim \Delta, A}{\Gamma, \neg A \mid\sim \Delta}$$

R¬: $$\frac{\Gamma, A \mid\sim \Delta}{\Gamma \mid\sim \Delta, \neg A}$$

L∧: $$\frac{\Gamma, A, B \mid\sim \Delta}{\Gamma, A \wedge B \mid\sim \Delta}$$

R∧: $$\frac{\Gamma \mid\sim \Delta, A \quad \Gamma \mid\sim \Delta, B \quad \Gamma \mid\sim \Delta, A, B}{\Gamma \mid\sim \Delta, A \wedge B}$$

L∨: $$\frac{\Gamma, A \mid\sim \Delta \quad \Gamma, B \mid\sim \Delta \quad \Gamma, A, B \mid\sim \Delta}{\Gamma, A \vee B \mid\sim \Delta}$$

R∨: $$\frac{\Gamma, \mid\sim \Delta, A, B}{\Gamma \mid\sim \Delta, A \vee B}$$

To make out that notion of reversible, I must explain **the relation between *derivable* rules and *admissible* metainferential rules for sequents**.
So whether or not S|~T can be derived from S'|~T'.
(from empty or from arbitrary leaves) using the rules, is it the case that whenever S'|~T' can be derived, so can S|~T.
Admissibility is a *metainferential* concept: whenever one conclusion can be derived (from the same premises), so can the other.
This is the beginning of the hierarchy of meta-inferences that the Buenos Aires folks study.
Can call a sequent calculus "structurally complete" in case **all *admissible* rules are *derivable***.
There are such calculi.

Must also talk about how we need to reconceptualize theorem in substructural settings. For in such settings, being able to derive S|~T from empty, flat, or RE, or CO leaves is one thing. Being able to derive it from *arbitrary* sequents is quite another. The notion of theorem that we need in the substructural cases is the latter.

Looking at how SC specifications of logic that are equivalent under closure structure diverge in expressive power when the structure is opened up.
Connective rules.
Why it is important, for metatheoretic arguments about what can and cannot be derived in an SC, that the rules only complicate.
but this seems to rule out the biconditionals essential to DD and II.
Cut and Hauptsatz.
Ketonen and reversible rules.
Q: How possible?
A: Distinguish derivability from admissibility.

That is what makes reversible rules possible, or more precisely, it defines the sense innwhich they are possible: whenever the sequents on one side of the vertical line are derivable, so are the sequents on the other side. This definition then gives definite sense to the biconditionals in DD and II.

We can show that the connective rules of NMMS conservatively extend arbitrary (CO compliant) base vocabularies. They let us compute the reason relations of arbitrary (sets of) sentences in the logically extended lexicon, from the reason relations (sequents) in the base vocabulary (whether it is flat or substantive).
So much for "elaborated from".
What about "explicative of"?

Dan's expressive completeness result.
The first half is that any set of base sequents can be associated with all the sets of logically complex sequents it makes good. because our rules can do this for *any* set of base sequents, NMMS is *expressively complete* in a sense analogous to Post-completeness for two-valued truth-tables. Post completeness says that *every possible* truth table is the truth-table of some logically complex sentence that can be formulated using $\rightarrow, \neg, \vee, \wedge$. We are codifying reason relations, not assignments of binary truth values, but the analogy is strong, for the implication/incompatibility species of the *basic discursive bipolarity*, rather than the true/false species.

Kaplan's expressive completeness representation theorem for NMMS (*RLLR* p. 130):
For any set *AtomicImp* of sequents in any base vocabulary L_B, there is a nonempty set of sequents *ExtImp* in the logical extension of L_B by NMMS such that every individual sequent in *ExtImp* is derivable if and only if all the sequents of *AtomicImp* hold. And conversely, for any set of sequents *ExtImp* in the logical extension of L_B by NMMS, we can compute the exact set *AtomicImp* of sequents defined on the lexicon of the base vocabulary that must hold in order for all the sequents in *ExtImp* to be derivable.

Fact: **NMMS is *universally* LX for all base vocabularies that satisfy CO.**

Kaplan expressive completeness theorem gives a definite sense in which logically complex sentences make sets of reason relations *explicit*, that is *say that* those relations hold.

So we have defined a sense in which we can prove that NMMS is universally LX: conservatively elaboratable from and explicative of any (CO-compliant) vocabulary whatsoever.

<u>Explicitly Marking Local Regions of Structure</u> <u>(*RLLR* section 3.3 pp. 131ff)</u>:
Can mark implications that hold *persistently* (monotonically), so that $\forall X \subseteq L[\Gamma, X |\!\sim A, \Delta]$, by "$\Gamma |\!\sim^{\uparrow} A, \Delta$." Then can introduce an operator to codify the persistence of that sequent as "$\Gamma |\!\sim \Box A, \Delta$." Similarly, we can make explicit local regions of *classicality* (MO + CT).

III. Trilogics K3 and LP from an Expressivist Perspective

Have so far mostly addressed problems and criteria of adequacy set by our own program (expressivism). Here is a conceptual redescription in our vocabulary of a familiar and well-studied issue, which has a remarkable outcome.

From a semantic point of view, Strong Kleene matrices for connectives, common to the logics K3 and LP:

¬	
1	0
½	½
0	1

⊃	**1**	**½**	**0**
1	1	½	0
½	1	½	½
0	1	1	1

∧	**1**	**½**	**0**
1	1	½	0
½	½	½	0
0	0	0	0

∨	**1**	**½**	**0**
1	1	1	1
½	1	½	½
0	1	½	1

K3 treats only 1 as designated (preserved by good implications).
LP treats both 1 and ½ as designated (preserved by good implications).
Thereby, K3 interprets ½ as "Neither", and LP interprets ½ as "Both."
K3 is the paracomplete logic of truth-value *gaps*.
LP is the paraconsistent logic of truth-value *gluts*.

K3 and LP are duals in that an inference is valid in one just in case its contrapositive is valid in the other. (But this fact doesn't engage with my story.)

Taking the underlying consequence relation to be (nonmonotonic and) nontransitive splits premissory and conclusory metainferential consequence relations=reason relations. Those new consequence relations are transitive. LP is can be nonmonotonic.

In addition to broadly inferential connections *across* the turnstile, there are *meta*inferential relations comparing premise-sets to premise-sets (based on what conclusions they lead to), and conclusion-sets to conclusion-sets (based on what premises lead to them). In each case we just use set-theoretic inclusions among those conclusion-sets to order premise-sets, or premise-sets to order conclusion-sets.
In full structural conditions, the implication p|~q settles it both that, q can always be substituted for p as a premise *salva consequentia* (our intensional inferentialist analogue to Quine's

paradigmatically extensional "salva *veritate*"), and that p can always be substituted for q as a conclusion, *salva consequentia*. In substructural settings, these conditions come apart. This is another instance, to be compared to SC specifications of the reason relations of classical logic that are all equivalent in structurally closed settings, but come apart in substructural or open-structured settings.

Construing the trilogics from an inferential point of view, in the presence of Cut (CT), and Monotonicity (MO), A|~B licenses, for arbitrary sets of sentences Γ,Δ, **A|~B** licenses:

i) Premissory substitution license:

<u>Γ,q|~Δ</u>
Γ,p|~Δ

ii) Conclusory substitution license:

<u>Γ|~p,Δ</u>
Γ|~q,Δ

<u>**Main Result:**</u>

K3's implication-codifying turnstile is of type (i), having the *meta*inferential significance of a *premissory* substitution license.
LP's implication-codifying turnstile is of type (ii), having the *meta*inferential significance of a *conclusory* substitution license.

Q: Why does the logic of premissory substitution metainferences also show up as the paracomplete logic of truth-value *gaps* and the logic of conclusory substitution metainferences also show up as the paraconsistent logic of truth-value *gluts*?
A: In the underlying substructural |~ relation in NMMS, it can happen that in the base vocabulary, for some sentences A and B transitivity fails in such a way that one cannot Cut on A∨¬A as a premise, or B∧¬B as a conclusion. In this sense, the logic of premissory substitutional metainferences does not *accept* some instances of Excluded Middle and the logic of conclusory substitutional metainferences does not *reject* some instances of Noncontradiction. In a semantic setting that construes consequence in terms of preservation of designated truth-values, these facts show up as gaps and gluts. (Cf. *RLLR* pp. 244-245).
K3 and LP are both fully transitive and monotonic.

We can drop $A \vee \neg A$ as a premise or,
equivalently, $A \wedge \neg A$ as a conclusion in an implication, *salva consequentia*,
just in case we can apply Cut to two implications that are like the one with which we started except that *A* figures as a premise in one and as a conclusion in the other.
The metainference in question is:

1. Γ,A|~A,Δ by CO
2. Γ|~¬A,A, Δ by ¬R
3. Γ|~¬A∨A,Δ by ∨R
4. Γ,¬A∨A|~Δ Assumption
5. Γ|~Δ. From (3) and (4), by CT ?????

But if one can't Cut on A, one can't Cut on ¬A∨A either. (Show this.)
The result is that ¬A∨A is not a "free" premise, in the sense of a premise one is always entitled to accept. That looks like an implicit *rejection* of ¬A∨A: a truth-value *gap*.

IV. Four examples of distinctions-with-relations emerging from relaxation of structure:

1. (Well-known:) One of Gentzen's astonishing accomplishments was to show that the very same set of connective definitions and structural constraints (MO, CT) specifies *intuitionistic* logic if one requires that sequents only have single formulae on the right ("single-succedent" sequents), and specifies *classical* logic if one relaxes that requirement and allows more than one formula on the right ("multi-succedent" sequents).

2. (New:) One large lesson of this week is that connective-defining sequent rules that specify the *same* logic under strong structural closure constraints (MO, CT), can come apart and determine different consequence relations and incompatibilities when those structural constraints are relaxed. Gentzen's LK rules and Ketonen's reversible rules both specify *classical* logic if MO and CT hold.

3. (Well-known:) Strong Kleene multivalue truth tables or matrices yield very different logics if they are conjoined with different notions of consequence. K3 and LP are alike in using preservation-of-designatedness of multivalues to define consequence, differing only in which multivalues are designated. The Strict-Tolerant logic ST shows a quite different way to define consequence.

4. (New:) We ended by comparing definitions of consequence *salva veritate* (which includes K3 and LP) vs. *salva consequentia*. Under fully closed structural conditions, if A|~B, then A can be substituted everywhere for B as a premise, and B can be substituted everywhere for A as a conclusion, saving the goodness of sequents. Defining consequence by considering substitutions *salva consequentia* (which substitutions of sentences for sentences preserves the goodness of reason relations) splits into two different metainferential relations, *premissory*-role inclusion and *conclusory*-role inclusion, if mixed context Cut (transitivity) fails.